Available online at www.elixirpublishers.com (Elixir International Journal)

Applied Mathematics

Elixir Applied Mathematics 167 (2022) 56321-56327

ISSN: 2229-712X

Spectral Decomposition of Matrix

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ARTICLE INFO

Article history:
Received: 5 May 2022;
Received in revised form: 19 June 2022;
Accepted: 29 June 2022;

Keywords
Matrix Analysis Methods,
Decomposition Method.

ABSTRACT
In this article, we study one of the matrix analysis methods, which is the spectral decomposition method of the matrix, which is a very special case for the square and symmetric real matrix.

Introduction

In this article, we study one of the matrix analysis methods, which is the spectral decomposition method of the matrix, which is a very special case for the square and symmetric real matrix. For the purpose of understanding how to analyze, we must understand several things :

Eigen value /Subjective values.
Eigenvectors
Processing of vectors using the Gram-schmidt Method
Gram-schmidt Method to be perpendicular to each other
Internal multiplication
Inner product (for vectors external beating)
outer product (for vectors).
Row echelon form (ref) for the matrix.
Reduce row echelon form (rref) for the matrix.
Rank of a matrix.
Row rank and column rank.

These concepts are from the core of advanced linear algebra, and they are all included in the symmetric square matrix spectroscopy lecture. Analysis is transforming the matrix into the product of three matrices so that we can convert it into a linear polynomial structure and each term participates in building the original matrix at a certain level, the largest of which is concentrated in the first terms and then decreases until we reach the last term...

For example, if the matrix is transformed into a linear structure of 1,000 terms, the first 50 terms of it can contribute to building 96% of the original matrix, and 950 terms contribute to building the remaining 4%. Which allows us to neglect the last 950 limits and stick to only the first 50 limits, and this topic is included in thousands of researches related to storing information or documents, document encryption, and other applications.

Of course, in this article we focus on a topic that paves the way for the most important topic, which is Singular Value Decomposition (SVD) which is more general than the topic of the lecture because it deals with all kinds of real matrices but with different and more complex constants.

The general objective of the article

Acquire the student the concept of matrix spectroscopy

Spectral decomposition of matrix

The experience around which the objective revolves (the main topic of the lesson):-

Spectral decomposition of matrix

That the student knows (Spectral decomposition of matrix)

1. The student determines the steps to be taken to perform an operation (Spectral decomposition of matrix).
2. To give the student some of the benefits of this analysis through an example.
3. Give an example of a symmetric square matrix that can be analyzed in this way.
4. The student gives an example of a matrix that cannot be analyzed in this way.

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5. The student performs this analysis on a real example of a symmetric square matrix.
6. The student determines the numerical benefit of this analysis.

Sub-topic elements

1-Eigen value (Self-value concept).
2-Eigen vector (The concept of eigenvectors).
3- Gram-Schmidt method (Processing of vectors using the Gram-schmidt Method to be perpendicular to each other.
4-Inner product (The concept of internal multiplication for vectors) .
5- Outer product (The concept of external multiplication for vectors).
6-Concept Row echelon form (ref) for the matrix
7-Concept Reduce row echelon form (rref) for the matrix
8-Concept Rank of a matrix
9-Concept Row rank and column rank

i) Definition (Spectral decomposition)

Lit A be an n×n symmetric matrix can be expressed as the matrix product:-

$$A = P * D * P^T \dots\dots\dots\dots\dots \quad (1)$$

D is diagonal matrix with entries are the eigenvalues of A.

P is an orthogonal matrix that its columns are associated orthonormal eigenvectors $x_1, x_2, \dots, x_n$.

The expression (1) is called (the spectral decomposition of A), and we can write it in the following form:-

$$A = [x_1 \quad x_2 \dots \quad x_n] \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix} \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix} \dots\dots\dots\dots \quad (2)$$

In particular, we can express A as a linear combination of simple symmetric matrices that are fundamental building blocks of that total information within A.

The product $D * P^T$ can be computed and gives:

$$D * P^T = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix} \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix} = \begin{bmatrix} \lambda_1 x_1^T \\ \lambda_2 x_2^T \\ \vdots \\ \lambda_n x_n^T \end{bmatrix}$$

And hence (2) becomes:

$$A = [x_1 \quad x_2 \dots \quad x_n] \begin{bmatrix} \lambda_1 x_1^T \\ \lambda_2 x_2^T \\ \vdots \\ \lambda_n x_n^T \end{bmatrix} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3)$$

So we can express A as a linear combination of the matrices x_j, x_j^T and the coefficients are the eigenvalues of A.

That is:-

$$A = \sum_{j=1}^{n} \lambda_j x_j x_j^T$$

$$A = \lambda_1 x_1 x_1^T + \lambda_2 x_2 x_2^T + \cdots + \lambda_n x_n x_n^T \dots\dots\dots\dots\dots\dots\dots \quad (4)$$

Lemma:-

Since P is orthonormal matrix

∴ P is orthogonal matrix

$$\Longrightarrow P^{-1} = P^T \Longrightarrow PP^T = I$$

∴ by multiple (**) by P^T for left and by P for right:

$$P^T * A * P = P^T * P * D * P^T * P = I * D * I = D$$

QED

Illustration of the example in general

Example (1)

Let A be a 2×2 symmetric matrix with eigenvalues λ_1 and λ_2 and associated orthonormal eigenvectors x_1 and x_2:

Proof in case (2×2)

Let $p = [x_1 \quad x_2]$, $x_1 = \begin{bmatrix} a \\ b \end{bmatrix}$ $and\ x_2 = \begin{bmatrix} c \\ d \end{bmatrix}$

$$A := \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} \lambda_1 a & \lambda_1 b \\ \lambda_2 c & \lambda_2 d \end{bmatrix}$$

$$= \begin{bmatrix} \lambda_1 a^2 + \lambda_2 c^2 & \lambda_1 ab + \lambda_2 cd \\ \lambda_1 ab + \lambda_2 cd & \lambda_1 b^2 + \lambda_2 d^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 a^2 & \lambda_1 ab \\ \lambda_1 ab & \lambda_1 b^2 \end{bmatrix} + \begin{bmatrix} \lambda_2 c^2 & \lambda_2 cd \\ \lambda_2 cd & \lambda_2 d^2 \end{bmatrix}$$

$$= \lambda_1 \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix} + \lambda_2 \begin{bmatrix} c^2 & cd \\ cd & d^2 \end{bmatrix} = \lambda_1 \begin{bmatrix} a[a & b] \\ b[a & b] \end{bmatrix} + \lambda_2 \begin{bmatrix} c[c & d] \\ d[c & d] \end{bmatrix}$$

$$= \lambda_1 \begin{bmatrix} a \\ b \end{bmatrix} [a \quad b] + \lambda_2 \begin{bmatrix} c \\ d \end{bmatrix} [c \quad d] = \lambda_1 x_1 x_1^T + \lambda_2 x_2 x_2^T = \sum_{j=1}^{2} \lambda_j x_j x_j^T$$

This example is for illustration in general assuming eigenvalues is orthonormal It serves as proof to demonstrate the linear structure (4) and can be circulated by mathematical induction.

The following example in which not orthonormal eigenvectors:-

Example (2)

Let

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

To determine the spectral representation of A, we first obtain its eigenvalues and eigenvectors.

We find that A has three distinct eigenvalues:

$\lambda_1 = 1 \quad , \quad \lambda_2 = 3 \quad \& \; \lambda_3 = -1$

And that associated eigenvectors are respectively verify:

$$x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad , \quad x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \& \quad x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Since the eigenvalues are distinct we are assured that corresponding eigenvectors form an orthogonal set. To normalizing these vectors, we obtain eigenvectors of unit length that are in orthonormal set:

$$x_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \Rightarrow \quad \|x_1\| = \sqrt{2}$$

$$x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \quad \|x_2\| = \sqrt{2}$$

$$x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \quad \|x_3\| = 1$$

An orthonormal set $\frac{1}{\sqrt{2}} x_1 \;,\; \frac{1}{\sqrt{2}} x_2 \;,\; x_3$

$\therefore new\ x_i \;\; (i = 1,2,3)$

$$x_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, x_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \text{ And } \quad x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Then the spectral representation of A is:-

$$A = \lambda_1 x_1 x_1^T + \lambda_2 x_2 x_2^T + \lambda_2 x_2 x_2^T$$

$$= (1)\left(\frac{1}{\sqrt{2}}\right)^2 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} [1 \quad -1 \quad 0] + (3)\left(\frac{1}{\sqrt{2}}\right)^2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} [1 \quad 1 \quad 0] + (-1) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [0 \quad 0 \quad 1]$$

$$= \left(\frac{1}{2}\right) \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (3)\left(\frac{1}{2}\right) \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (-1) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + (3) \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + (-1) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ldots\ldots (\#)$$

For test

(#) is equal:-

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} & \frac{3}{2} & 0 \\ \frac{3}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A$$

If the symmetric matrix has repeated eigenvalue then the set of the eigenvectors dos not orthogonal.

However, we can apply the Gram-schmidt process to the linearly independent eigenvectors associated with a repeated eigenvalue to obtain a set of orthogonal eigenvectors.

The Gram-schmidt process:-

We define the projection operator by:

$$proj_u(v) = \frac{\langle u.v\rangle}{\langle u.u\rangle} u$$

Where $\langle u.v\rangle$ denotes the inner product of the vectors u and v.

This operators projects the vector v orthogonally onto the line spanned by vector u.

The Gram-schmidt process then works as follows:

Let $\{v_1, v_2, \dots, v_k\}$ be collection of not orthogonal vectors.

$$u_1 = v_1 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots e_1 = \frac{u_1}{\|u_1\|}$$

$$u_2 = v_2 - proj_{u_1}(v_2) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots e_2 = \frac{u_2}{\|u_2\|}$$

$$u_3 = v_3 - proj_{u_1}(v_3) - proj_{u_2}(v_3) \dots\dots\dots\dots\dots\dots\dots\dots e_3 = \frac{u_3}{\|u_3\|}$$

$$u_4 = v_4 - proj_{u_1}(v_4) - proj_{u_2}(v_4) - proj_{u_3}(v_4) \dots\dots e_4 = \frac{u_4}{\|u_4\|}$$

$$u_k = v_k - \sum_{j=1}^{k-1} proj_{u_j}(v_k) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots e_k = \frac{u_k}{\|u_k\|}$$

The sequence $u_1, u_2, \dots, u_k$ is the required system of orthogonal vectors.

The normalized vectors $e_1, e_2, \dots, e_k$ form an orthonormal set.

The calculation of the sequence $u_1, u_2, \dots, u_k$ is known Gram-schmidt orthogonalization the calculation of the sequence $e_1, e_2, \dots, e_k$ is known as Gram-schmidt orthonormalization as the vectors normalized.

And, of course, possible to make sure that the $\langle u_i . u_k\rangle = 0$ for all i, k which is confirms that the vectors u_i *for all* i are orthogonal.

Example (3) for Gram-schmidt process.

Let $v_1 = (1,-1,2)$, $v_2 = (0,2,-1)$, $v_3 = (-1,1,1)$

$v_1, v_2 \& v_3$ are not orthogonal vectors.

By Gram-schmidt:-

$$u_1 = v_1 = (1,-1,2)$$

$$u_2 = v_2 - proj_{u_1}(v_2) = (0,2,-1) - \frac{\langle u_1 . v_2\rangle}{\langle u_1 . u_1\rangle} u_1$$

$$= (0,2,-1) - \frac{(1\times 0 - 1\times 2 + 2\times(-1))}{(1+1+4)}(1,-1,2) = (\frac{2}{3}, \frac{4}{3}, \frac{1}{3})$$

$$u_3 = v_3 - proj_{u_1}(v_3) - proj_{u_2}(v_3)$$

$$= (-1,1,1) - \frac{\langle u_1 . v_3\rangle}{\langle u_1 . u_1\rangle} u_1 - \frac{\langle u_2 . v_3\rangle}{\langle u_2 . u_2\rangle} u_2$$

$$= (-1,1,1) - \frac{(-1-1+2)}{(1+1+4)}(1,-1,2) - \frac{\left(\frac{-2}{3} + \frac{4}{3} + \frac{1}{3}\right)}{\left(\frac{4}{9} + \frac{16}{6} + \frac{1}{6}\right)}\left(\frac{2}{3}, \frac{4}{3}, \frac{1}{3}\right)$$

$$= (\frac{-9}{7}, \frac{3}{7}, \frac{6}{7})$$

$$e_1 = \frac{u_1}{\|u_1\|} = \frac{1}{\sqrt{6}}(1,-1,2)$$

$$e_2 = \frac{u_2}{\|u_2\|} = \sqrt{\frac{3}{7}}(\frac{2}{3}, \frac{4}{3}, \frac{1}{3})$$

$$e_3 = \frac{u_3}{\|u_3\|} = \sqrt{\frac{7}{18}}(\frac{-9}{7}, \frac{3}{7}, \frac{6}{7})$$

$$u_1 . u_2 = \frac{2}{3} - \frac{4}{3} + \frac{2}{3} = 0$$

$$u_1.u_3 = \frac{-9}{7} - \frac{3}{7} + \frac{12}{7} = 0$$

$$u_2.u_3 = \frac{-18}{21} + \frac{12}{21} + \frac{6}{21} = 0$$

$\therefore\ u_1, u_2 \& u_3$ are orthogonal vectors $and\ e_1, e_2 \& e_3$ are orthonormal vectors.

General example (4) for inner product:-

Let $A = (a_1, a_2, \dots, a_n)\ and\ B = (b_1, b_2, \dots, b_n)$

The inner product between A and B:

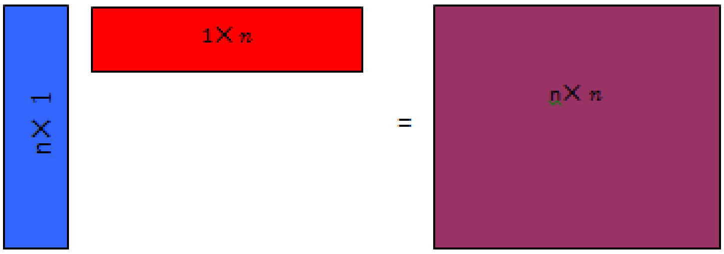
$$A.B = \sum_{i=1}^{n} a_i b_i \quad if\ a_i\ and\ b_i\ are\ real\ numbers$$

Or $A.B = \sum_{i=1}^{n} \overline{a_i} b_i \quad if\ a_i\ and\ b_i\ are\ comblex\ numbers$

The formula (4) is private for symmetric matrix and it's a linear combination of matrices $x_j x_j^T$ which are $n \times n$, since x_j is $n \times 1$ and x_j^T is $1 \times n$.

The matrix $x_j x_j^T$ is simple construction.

$x_j \times x_j^T =$



$x_j \times x_j^T$ is called outer product

2-Definition (outer product)

Let X and Y are $n \times 1$ matrices whose entries are $x_1, x_2, \dots, x_n$ and $y_1, y_2, \dots, y_n$ respectively.

$$i.e: X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \ ,\ Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

The outer product of X and Y is the matrix product XY^T which gives the $n \times n$ matrix:

$$XY^T = \begin{bmatrix} x_1y_1 & x_1y_2 & \cdots & x_1y_n \\ x_2y_1 & x_2y_2 & \cdots & x_2y_n \\ \vdots & \vdots & \cdots & \vdots \\ x_ny_1 & x_ny_2 & \cdots & x_ny_n \end{bmatrix}$$

Example(5)/form the outer product of X and Y where:

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\ and\ Y = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Solution:

$$XY^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [4 \quad 5 \quad 6] = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$$

Not.: each row in $x_j x_j^T$ matrix is a multiple of x_j^T

3-Definitions (Row echelon form (ref)

A matrix is in row echelon form (ref) when it satisfies the following conditions:-

i) The first non-zero element in each row called the leading entry is 1.

ii) Each leading entry is in a column to the right of the leading entry in the previous row.

iii) Rows with all zero elements if any are below rows having a non-zero element.

4-Definition (Reduce row echelon form (ref)

A matrix is in rref when it satisfies the following conditions:-

i)The matrix is in row echelon form (it satisfies the three conditions listed above).

ii) The leading entry in each row is the only non-zero entry in it's column.

Example(6)

$$The\ matrix\ A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\ is\ in\ ref$$

$$And\ the\ matrix\ B = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\ is\ in\ rref$$

5-Definition (Rank of a matrix)

Rank of a matrix A is equal the number of non-zero rows in rref(A).

Important results:-

i) The set of all non-zero rows in rref(A) is equal the bases of the row spaces of the matrix A.

ii)The number of non-zero rows in rref(A) is equal the number of columns that containing the leading 1's.

6-Definition (Row rank and column rank)

The dimension of the row space of A is called the row rank of A and the dimension of the column space of A is called the column rank of A.

Since the row rank and the column rank of a $m \times n$ matrix A are equal we only refer to the rank of A and writ rank (A).

Example (7)

let A be another $n \times n$ matrix and

$$rref(A) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Since\ \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}\ is\ a\ basis\ of\ column\ space\ and\ thus$$

$$rank(A) = 3$$

The solutions of $AX = 0$ are

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = s_1, x_5 = s_2\ ,and\ s_1, s_2 \in \mathbb{R}$$

Hence the reduced row echelon form of $x_j x_j^T$ $\left[denoted\ rref(x_j x_j^T)\right]$ has one non-zero row and thus has rank one, we interpret this in (4) to mean that each outer product $x_j x_j^T$ contributes just one piece of information to the construction of matrix A.

To pave for applications:

From example 2 we have that the eigenvalues are ordered as:

$|3| \geq |-1| \geq |1|$. Thus the contribution of the eigenvectors corresponding to eigenvalues -1 and 1 can be considered equal, whereas that corresponding to 3 is dominant.

Rewriting the spectral decomposition using the terms eigenvalues, we obtain the following:

$$A = 3\left(\frac{1}{\sqrt{2}}\right)^2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} + (-1) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} + 1\left(\frac{1}{\sqrt{2}}\right)^2 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$$

$$= 3\left(\frac{1}{2}\right) \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (-1) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 1\left(\frac{1}{2}\right) \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Looking at the terms of the partial sums in this example individually.

We have the following matrices:

i) $\begin{bmatrix} \frac{3}{2} & \frac{3}{2} & 0 \\ \frac{3}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

ii) $\begin{bmatrix} \frac{3}{2} & \frac{3}{2} & 0 \\ \frac{3}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{3}{2} & 0 \\ \frac{3}{2} & \frac{3}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix}$

iii) $$\begin{bmatrix} \frac{3}{2} & \frac{3}{2} & 0 \\ \frac{3}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & 0 \\ \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

This suggests that we can approximate the information in the matrix A, using the partial sums of the spectral decomposition.
In fact, this type of development is the foundation of a number of approximation procedures in mathematics.

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