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Generalized Exponential Type Estimators under Simple Random Sampling and Qualitative Auxiliary Information

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ABSTRACT

The primary aim of the present work is to prepare a generalized exponential type estimators for estimation of the population mean under simple random sampling and qualitative auxiliary information. The expressions for MSE have been obtained. A comparison between proposed estimators with some existing estimators is done theoretically as well as numerically.

Introduction

Consider a finite population which consists of N identifiable units $\Omega_i\,(1 \le i \le N)$. Assume that a sample of size n drawn by using simple random sampling without replacement (SRSWOR) from a population of size N. Let y_i and θ_i denote the observations on the variable y and θ respectively for i^{th} unit (i=1, 2,......, N). Suppose there is a complete dichotomy in the population with respect to the presence or absence of an attribute, say θ, and it is assumed that attribute θ takes only two values 0 and 1 according as $\theta_i = 1$ if i^{th} unit of the population possesses attribute θ otherwise zero. Let $K = \sum_{i=1}^{N}\theta_i \big/ N$ and $k = \sum_{i=1}^{n}\theta_i \big/ n$ denote the proportion of units in the population and sample respectively possessing attribute θ. Let $\bar{y} = \frac{1}{n}\sum_{i}^{n} y_i$ and $k = \frac{1}{n}\sum_{i}^{n}\theta_i$ be the sample means of variable of interest y and auxiliary attribute θ and $\bar{Y} = \frac{1}{N}\sum_{i}^{N} y_i$ be the corresponding population means. We take the situation when the mean of the auxiliary attribute (K) is known. Let $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ and $s_\theta^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\theta_i - k)^2$ be the sample variance and $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$ and $S_\theta^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\theta_i - K)^2$ be the corresponding population variance. Let $\rho_{y\theta} = S_{y\theta} \big/ S_y S_\theta$ be the point bi-serial correlation coefficient between y and θ. Finally we define, $\Lambda_y = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ *and* $\Lambda_\theta = \frac{k - K}{K}$ such that $E\,[\Lambda_i] = 0$ *for* $(i = y, \theta)$, $E(\Lambda_y^2) = \zeta\, S_y^2 \big/ \bar{Y}^2$, $E(\Lambda_\theta^2) = \zeta\, S_\theta^2 \big/ K^2$ and $E(\Lambda_y \Lambda_\theta) = \zeta\, \rho_{y\theta}\, S_y \big/ \bar{Y}\; S_\theta \big/ K$ where $\zeta = \left(\frac{1}{n} - \frac{1}{N}\right)$. The terms defined above are used to determine the characteristic of the proposed estimator and existing estimators considered here.

Some Existing Estimators

In this section, we consider the several existing estimators under simple random sampling and qualitative auxiliary information which are used for the estimation of population mean.

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Table 1. Some Exiting estimators using Qualitative Auxiliary Information

Estimator	**Mean Square Error [MSE(*)]**
Ratio Estimator by Bahl and Tuteja (1991) $\Theta_1 = \bar{y}\, \exp\left(\frac{K-k}{K+k}\right)$	$MSE(\Theta_1) = \zeta\ \bar{Y}^2\left[C_y^2 + \frac{1}{4}C_\theta^2 - \rho_{y\theta}\, C_y C_\theta\right]$
Product Estimator by Bahl and Tuteja (1991) $\Theta_2 = \bar{y}\, \exp\left(\frac{k-K}{k+K}\right)$	$MSE(\Theta_2) = \zeta\ \bar{Y}^2\left[C_y^2 + \frac{1}{4}C_\theta^2 + \rho_{y\theta}\, C_y C_\theta\right]$
Ratio Estimator by Naik and Gupta (1996) $\Theta_3 = \left(\frac{\bar{y}}{k}\right)K$	$MSE(\Theta_3) = \zeta\ \left[S_y^2 + R^2 S_\theta^2 - 2R\rho_{y\theta} S_y\, S_\theta\right]$
Product Estimator by Naik and Gupta (1996) $\Theta_4 = \left(\frac{\bar{y}}{K}\right)k$	$MSE(\Theta_4) = \zeta\ \left[S_y^2 + R^2 S_\theta^2 - 2R\rho_{y\theta} S_y\, S_\theta\right]$

Proposed Estimator and their Properties

In this section, generalized exponential type estimator for the estimation of population mean is proposed under same sampling scheme defined in section (2) as:

$$\Theta_5 = [\bar{y} - (e^{\Delta_1} - 1)] \quad (3.1)$$

$$\Theta_6 = [\bar{y} - (e^{\Delta_2} - 1)] \quad (3.2)$$

where $\Delta_1 = \left[\frac{k-K}{k+K}\right]$ and $\Delta_2 = \left[\frac{K-k}{K+k}\right]$.

To obtain the properties of the proposed estimators, expend the equation (3.1) and (3.2) in terms of $\Lambda's$.

For Θ_5, expanding the right hand side of equation (3.1) up to the second order of approximation in terms of $\Lambda's$, we have

$$\Theta_5 = \bar{y}(1+\Lambda_y) - \exp[\frac{\Lambda_\theta}{2+\Lambda_\theta}] + 1$$

$$\Theta_5 = \bar{y}(1+\Lambda_y) - \frac{\Lambda_\theta}{2}(1 - \frac{\Lambda_\theta}{2}) - \frac{\Lambda_\theta^2}{8}$$

Taking expectation on both sides, we get

$$E(\Theta_5) = \bar{Y} + E\ [\frac{\Lambda_\theta^2}{8}]$$

So Θ_5 is a biased estimator of the population mean $\bar{Y}$

$$Bias\ (\Theta_5) = \frac{1}{8}\zeta(\frac{S_\theta^2}{K^2})$$

Now

$$MSE\ (\Theta_5) = E[\Theta_5 - E(\Theta_5)]^2$$

$$MSE\ (\Theta_5) = E[\bar{Y}\Lambda_y - \frac{\Lambda_\theta}{2}]^2$$

$$MSE\ (\Theta_5) = \bar{Y}\ E[\Lambda_y^2] + \frac{1}{4}E[\Lambda_\theta^2] - \bar{Y}\ E[\Lambda_y \Lambda_\theta]$$

After simplification, mean square of Θ_5 is

$$MSE\ (\Theta_5) = \zeta\ [S_y^2 + \frac{1}{4}\ \frac{S_\theta^2}{K^2} - \frac{\rho_{y\theta} S_y S_\theta}{K}] \quad (3.3)$$

Similarly, right hand side of equation (3.2), expend up to the second order of approximation in terms of $\Lambda's$

$$\Theta_6 = \overline{y}(1+\Lambda_y) - \exp[\frac{-\Lambda_\theta}{2+\Lambda_\theta}]+1$$

$$\Theta_6 = \overline{y}(1+\Lambda_y) + \frac{\Lambda_\theta}{2}(1-\frac{\Lambda_\theta}{2}) - \frac{\Lambda_\theta^2}{8}$$

Taking expectation on both sides, we get

$$E(\Theta_6) = \overline{Y} - E\,[\frac{3\Lambda_\theta^2}{8}]$$

So Θ_6 is a biased estimator of the population mean $\overline{Y}$

$$Bias\ (\Theta_6) = -\frac{3}{8}\zeta(\frac{S_\theta^2}{K^2})$$

Now

$$MSE\ (\Theta_6) = E[\Theta_6 - E(\Theta_6)]^2$$

$$MSE\ (\Theta_6) = E[\overline{Y}\Lambda_y + \frac{\Lambda_\theta}{2}]^2$$

$$MSE\ (\Theta_6) = \overline{Y}\,E[\Lambda_y^2] + \frac{1}{4}E[\Lambda_\theta^2] + \overline{Y}\,E[\Lambda_y\Lambda_\theta]$$

After simplification mean square of Θ_6 is

$$MSE\ (\Theta_6) = \zeta\ [S_y^2 + \frac{1}{4}\frac{S_\theta^2}{K^2} + \frac{\rho_{y\theta}S_yS_\theta}{K}] \tag{3.4}$$

Efficiency of Estimators

Now we compare the proposed estimators Θ_5 and Θ_6 with existing estimators defined in section 2. We derive the following condition in which proposed estimators are better than the existing estimators:

Observation (i): Proposed Estimators vs. Ratio Estimator by Bahl and Tuteja (1991)

$$MSE(\Theta_1) - MSE(\Theta_5) > 0 \quad if \quad \rho_{y\,\theta} < \frac{S_\theta}{4\overline{Y}KS_y}\left(\overline{Y}+1\right)$$

$$MSE(\Theta_1) - MSE(\Theta_6) > 0 \quad if \quad \rho_{y\,\theta} < \frac{S_\theta}{4\overline{Y}KS_y}\left(\overline{Y}-1\right)$$

Observation (ii): Proposed Estimators vs. Product Estimator by Bahl and Tuteja (1991)

$$MSE(\Theta_2) - MSE(\Theta_5) > 0 \quad if \quad \rho_{y\,\theta} > \frac{S_\theta}{4\overline{Y}KS_y}\left(\overline{Y}-1\right)$$

$$MSE(\Theta_2) - MSE(\Theta_6) > 0 \quad if \quad \rho_{y\,\theta} > \frac{S_\theta}{4\overline{Y}KS_y}\left(\overline{Y}+1\right)$$

Observation (iii): Proposed Estimators vs. Ratio Estimator by Naik and Gupta (1996)

$$MSE(\Theta_3) - MSE(\Theta_5) > 0 \quad if \quad \rho_{y\,\theta} < \frac{S_\theta}{4KS_y}\left(2\overline{Y}+1\right)$$

$$MSE(\Theta_3) - MSE(\Theta_6) > 0 \quad if \quad \rho_{y\,\theta} < \frac{S_\theta}{4KS_y}\left(2\overline{Y}-1\right)$$

Observation (iv): Proposed Estimators vs. Product Estimator by Naik and Gupta (1996)

$$MSE(\Theta_4) - MSE(\Theta_5) > 0 \quad if \quad \rho_{y\,\theta} > \frac{S_\theta}{4KS_y}\left(2\overline{Y}-1\right)$$

$$MSE(\Theta_4) - MSE(\Theta_6) > 0 \quad if \quad \rho_{y\,\theta} > \frac{S_\theta}{4KS_y}\left(2\overline{Y}+1\right)$$

Numerical Study

In this section we compare the performance of various estimators considered here using the data sets as previously used by Shabbir and Gupta (2010).

Population: (Source: Sukhatme and Sukhatme (1970), pp. 256).

y = Number of villages in the circles, $\theta =$ A circle consisting more than five villages.

Table 2. Values of Parameters

$N = 89$ $\bar{Y} = 3.36$ $K = 0.124$ $\rho_{y\theta} = 0.766$ $n = 23$ $C_y = 0.604$ $C_\theta = 2.19$

Table 3. Percent Relative Efficiency

Estimators	**Percent Relative Efficiency w.r.t to Existing Estimators**			
	w.r.t Θ_1	**w.r.t. Θ_2**	**w.r.t. Θ_3**	**w.r.t. Θ_4**
Θ_1	100	*	*	*
Θ_2	*	100	*	*
Θ_3	*	*	100	*
Θ_4	*	*	*	100
Θ_5	124	379	472	973
Θ_6	114	350	438	899

Conclusion

In section 3, proposed modified exponential type estimators and also obtained the characteristic of the proposed estimators. In section 4, we derive the general conditions in which we can say proposed estimators are always more efficient than the existing estimators defined in section 2. In section 5, numerical study is done using the data used by Shabbir and Gupta (2010). In numerical study, we see that the proposed estimators have highest percent relative efficiency w.r.t to all existing estimators defined in section 2.

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