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Chemical and Radiation Effect on an Unsteady MHD Casson Fluid flow Passed over an Inclined Plate

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ABSTRACT

An unsteady MHD casson fluid flow past an inclined moving plate is examined to address the effect of chemical reaction and thermal radiation. The resulting system of the equations governing the flow is solved analytically using regular perturbation technique. The numerical results obtained are presented graphically against the different values of the parameters entering into the problem and interpreted physically. It is found that the results obtained in the present work are in excellent agreement with the physical reality of the problem.

Introduction

MHD convective flow problems together with heat and mass transfer have attracted the attention of a number of scholars because of their possible applications in many branches of science and technology. Geophysics encounters MHD characteristics in the interactions of conducting fluids and magnetic fields. Engineers employ MHD principle, in the design of heat exchangers pumps and flow meters, in space vehicle propulsion, thermal protection, braking, control and re-entry, in creating novel power generating systems etc. From technological point of view, MHD convection flow problems are also very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. MHD is also in stabilising a flow against the transition from laminar to turbulent flow and in reduction of turbulent drag and suppression of flow separation. The application of MHD principles in medicine and biology are of paramount interest owing to their significance in bio medical engineering in general and in the treatment of various pathological state in particular. Applications in biomedical engineering include cardiac MRI, ECG etc. Many have made model studies of the above phenomena of MHD convection. Some of them are Sanyal and Bhattacharya [2], Ferraro and Plumpton [1] and Cramer and Pai [3]. On the other hand, along with free convection currents, caused by the temperature difference, the flow is also affected by the difference in concentrations on material constitutions. Many investigators have studied the phenomena of MHD free convection and mass transfer flow of whom the names of Singh and Singh [4] and Singh *et al*. [5] are worth mentioning.

The analysis of boundary layer flow of viscous and non-Newtonian fluids has been the focus of extensive research by various scientists due to its importance in continuous casting, paper production, glass blowing, polymer extrusion and several others. Convective heat transfer plays an important role during the handling and processing of non-Newtonian fluid flows. Mechanics of non-Newtonian fluid flows present a special challenge to engineers, physicists and mathematicians. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. Study of Non-Newtonian fluid flow by Nakayama and Koyama [6], Banerjee et al.[7], Vedavathi et al.[8] are considered in this paper.

It is well known that Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear, i.e., if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move. Few examples of Casson fluid are: jelly, tomato sauce, honey, soup, concentrated fruit juices, etc. Many investigators have carried out model studies on Casson fluid flow; some of them are Shehzad et al.[9], Mukhophadhyay [10], Mukhophadhyay and Vajravelu [11], Dash et al. [12], Pushpalatha et al. [13].

The radiative effects have important applications in physics and engineering processes. The radiations due to heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends to some extent to the heat controlling factors. High temperature plasmas, liquid metal fluids, cooling of nuclear reactors and power generation systems are some important

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applications of radiative heat transfer from a vertical wall to conductive gray fluids.Comprehensive literature on various aspects of convective radiative MHD flows and its applications can be found in Sattar and Maleque [14], Samad and Rahman [15], Orhan and Ahmet [16], Prasad et al.[17], Takhar et al. [18], Ahmed and Sarmah [19] and Ahmed [20].

Contributions from many researchers on the study of Casson fluid flow in different geometry under the effect of different physical parameters viz. chemical reaction and thermal radiation etc are worth mentioning. Our aim is to investigate the effects of the various physical parameters involved in the problem on the fluid velocity and skin friction which are very important from the industrial application points of view.

Mathematical Analysis

Let us consider the unsteady MHD natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, thermal radiative and chemically reactive Casson fluid past over an oscillating inclined plate. Coordinate system is chosen in such a way that x'-axis is considered in vertically upward direction and y'-axis is in horizontal direction. The temperature and concentration on the plate oscillates with small amplitude about a non-uniform mean temperature. The fluid is assumed to have constant properties except for the influence of the density variations with temperature and concentration, which are considered only in the body force term. A uniform magnetic field is applied normal to the plate. It is also assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one.

In view of the above assumptions and taking into account the rheological equation for an incompressible and isotropic Casson fluid is

$$\tau = \tau_0 + \mu\alpha^*$$

Equivalently,

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, \pi < \pi_c \end{cases}$$

where τ, τ_0, μ and α^* are, respectively shear stress, Casson yield stress, dynamic viscosity and shear rate and $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i, j)th component of deformation rate, π is the product of component of deformation rate with itself, π_c is the critical value of this product based on the non-Newtonian model, μ_B is the plastic dynamic viscosity and p_y denote the yield stress of the fluid. Keeping in view the assumptions made above and usual Boussinesq's approximation, the governing equations are

$$\frac{\partial u'}{\partial t'} = \upsilon\left(1+\frac{1}{\alpha}\right)\frac{\partial^2 u'}{\partial y'^2} + g\cos\phi\beta\left(T'-T'_\infty\right) + g\cos\phi\beta^*\left(C'-C'_\infty\right) - \frac{\sigma}{\rho}B_0{}^2u' \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho Cp}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho Cp}\frac{\partial q'_r}{\partial y'} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr'\left(C'-C'_\infty\right) \quad (3)$$

with relevant boundary conditions:

$$u' = U,\ T' = T'_w + \varepsilon e^{i\omega t}\left(T'_w - T'_\infty\right),\ C' = C'_w + \varepsilon e^{i\omega t}\left(C'_w - C'_\infty\right) \text{ at } y' = 0 \quad (4a)$$

$$u' \to 0,\ T' \to 0,\ C' \to 0 \text{ as } y' \to \infty \quad (4b)$$

where u', g, α, ρ, β, β^*, k, C_p, σ, υ, D, T', C', Kr', q'_r and B_0 are, respectively, the fluid velocity in the x'-direction, acceleration due to gravity, Casson fluid parameter, the fluid density, the volumetric coefficient of thermal expansion, the volumetric coefficient of expansion for mass transfer, thermal conductivity, specific heat at constant pressure, electrical conductivity, the kinematic viscosity, the coefficient of mass diffusivity, the temperature of the fluid, species concentration, chemical reaction parameter, radiative heat flux vector and heat transfer coefficient.

The thermal radiation flux gradient may be expressed as follows

$$-\frac{\partial q_r'}{\partial y'} = 4a\sigma^*\left(T_\infty' - T_4'\right) \tag{5}$$

We assume that the temperature differences within the flow is sufficiently small, hence T'^4 may be expanded into Taylor's series about the free stream temperature (neglecting second and higher order terms) as follows:

$$T'^4 \approx 4T_\infty'^3 T' - 3T_\infty'^4$$

We now introduce the following non-dimensional quantities:

$$y = \frac{y'U}{\upsilon},\ u = \frac{u'}{U},\ t = \frac{t'U^2}{\upsilon},\ \theta = \frac{T' - T_\infty'}{T_w' - T_\infty'},\ \phi = \frac{C' - C_\infty'}{C_w' - C_\infty'},\ Gr = \frac{g\beta\upsilon\left(T_w' - T'\right)_\infty}{U^3},\ Gm = \frac{g\beta^*\upsilon\left(C_w' - C_\infty'\right)}{U^3},$$

$$Pr = \frac{\mu Cp}{k},\ Sc = \frac{\upsilon}{D},\ M = \frac{\sigma B_0^{\,2}\upsilon}{\rho U^2},\ R = \frac{16a\sigma^*\upsilon^2 T_\infty'^3}{U^2\rho C_p},\ Kr = \frac{\upsilon Kr'}{U^2}$$

where Gr, Gm, Pr, Sc, Kr, M and R are, respectively the Grashof number for heat transfer, the Grashof number for mass transfer, the Prandtl number, the Schmidt number, chemical reaction parameter, Hartmann number and radiation parameter.

The non-dimensional forms of the equations (1) to (3) are:

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\alpha}\right)\frac{\partial^2 u}{\partial y^2} + G_1\theta + G_2\phi - Mu \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - R\theta \tag{7}$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - Kr\phi \tag{8}$$

Here, $G_1 = Gr\cos\phi,\ G_2 = Gm\cos\phi$

The corresponding boundary conditions in non-dimensional form

$$u = 1,\ \theta = 1 + \varepsilon e^{i\omega t},\ \phi = 1 + \varepsilon e^{i\omega t} \quad \text{at } y = 0 \tag{9a}$$

$$u \to 0,\ \theta \to 0,\ \phi \to 0 \quad \text{as } y \to \infty \tag{9b}$$

Method of Solutions

We assume the solutions to be of the for

$$u = u_0 + \varepsilon e^{i\omega t} u_1 + 0\left(\varepsilon^2\right)$$

$$\theta = \theta_0 + \varepsilon e^{i\omega t}\theta_1 + 0\left(\varepsilon^2\right)$$

$$\phi = \phi_0 + \varepsilon e^{i\omega t}\phi_1 + 0\left(\varepsilon^2\right)$$

Substituting these in the equations (6) to (8) and by equating the co-efficient of the harmonic terms and neglecting ε^2 the following ordinary differential equations are obtained.

$$\left(1+\frac{1}{\alpha}\right)u_0'' - Mu_0 = -G_1\theta_0 - G_2\phi_0$$

$$\left(1+\frac{1}{\alpha}\right)u_1'' - (M+i\omega)u_1 = -G_1\theta_1 - G_2\phi_1$$

$$\frac{1}{Pr}\theta_0'' = R\theta_0$$

$$\frac{1}{Pr}\theta_1'' = (R+i\omega)\theta_1$$

$$\phi_0'' - Sc\,Kr\,\phi_0 = 0$$

$$\phi_1'' - Sc(Kr+i\omega)\phi_1 = 0$$

The boundary conditions are:

$$y=0: u_0=1,\ \theta_0=1,\ \phi_0=1, u_1=0, \theta_1=1,\ \phi_1=1$$

$$y\to\infty: u_0=0,\ \theta_0=0,\ \phi_0=0,\ \theta_1=0,\ \phi_1=0$$

The solutions of the above equations subject to the boundary conditions are

$$\theta = e^{-a_1 y} + \varepsilon e^{i\omega t} e^{-a_3 y}$$

$$\phi = e^{-a_2 y} + \varepsilon e^{i\omega t} e^{-a_4 y}$$

$$u = B_3 e^{-a_5 y} - B_1 e^{-a_1 y} - B_2 e^{-a_2 y} + \varepsilon e^{i\omega t}\left(B_6 e^{-a_6 y} - B_4 e^{-a_3 y} - B_5 e^{-a_4 y}\right)$$

The coefficient of skin friction (τ), rate of heat transfer in terms of Nusselt number (Nu) and rate of mass transfer in terms of Sherwood number (Sh) at the plate are:

$$\tau = -\left[\left(1+\frac{1}{\alpha}\right)\frac{\partial u}{\partial y}\right]_{y=0},\quad Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0},\quad Sh = -\left(\frac{\partial\phi}{\partial y}\right)_{y=0}$$

The constants $a_1, a_2, a_3, a_4, a_5, a_6, B_1, B_2, B_3, B_4, B_5, B_6$ are not shown here for the sake of brevity.

Results and Discussion:

In order to get physical insight into the problem, we have carried out the numerical calculations for the velocity, temperature, concentration fields, skin friction, Nusselt number and Sherwood number at the plate by assigning numerical values to the various parameters and variables involved in the problem. Our investigation is restricted to Prandtl number P=0.71, which corresponds to air at 20^0 C. The value of the Grashof number G_r for heat transfer has been chosen as 5 whereas the value of Grashof number G_m for mass transfer is considered to be 5. The values of the Schmidt number S_c are taken as .22 which corresponds to H_2 with air as the diffusing medium. In our investigation the values of the parameters ε, ϕ and ω are fixed at 0.0001, $\frac{\pi}{6}$ and 1 respectively. The values of the other parameters viz R the radiation parameter Kr the chemical reaction parameter and M the Hartmann number are chosen arbitrarily.

Figure 1 demonstrates the temperature profile against y under the influence of the Radiation parameter. Here it is observed that the fluid temperature falls asymptotically as y increases. The same figure also indicates that there is a steady drop of temperature due to radiation.

The variation of species concentration versus y is shown in figure 2. It is inferred from this figure that for increase in the value of the Kr, φ asymptotically falls as $y \rightarrow \infty$. That is the chemical reaction has a retarding effect on the concentration.

The velocity profiles against y are presented in figures 3-5 for different values of chemical reaction parameter, radiation parameter and magnetic parameter. These figures indicate that an increase in each of the values of Kr and M causes the boundary layer velocity to decrease. Within a thin layer adjacent to the plate the velocity gets accelerates due to thermal radiation, however the velocity decreases as the increase in the value of radiation parameter when it is observed far away from the plate. In other words the fluid motion is retarded due to these parameters and this phenomenon clearly supports the physical realities.

The variation of skin friction τ at the plate y = 0 against Kr and R are demonstrated in figures 6 and 7. These two figures indicate significant effect on the viscous drag that due to the application of the chemical reaction and radiation parameter relative to the applied magnetic field. It is contingent from these figures that near the plate the chemical reaction and radiation effect causes the magnitude of τ to decrease but as the magnetic effect increases or under the effect of magnetic field | τ | is increased slowly and steadily.

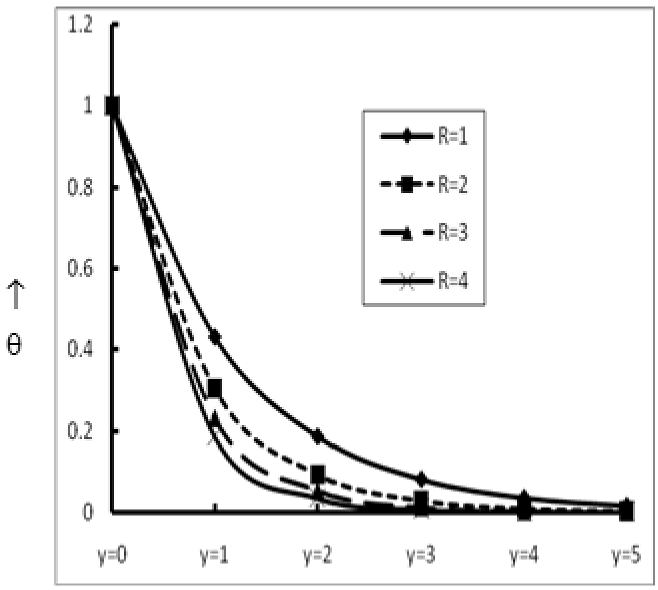

Fig 1: Temperature profile versus y for different values of Radiation Parameter

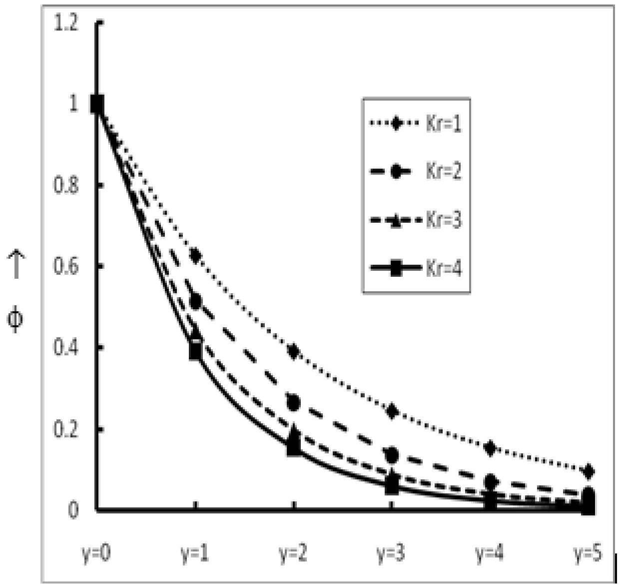

Fig 2: Concentration profile versus y for different values of Chemical reaction Parameter

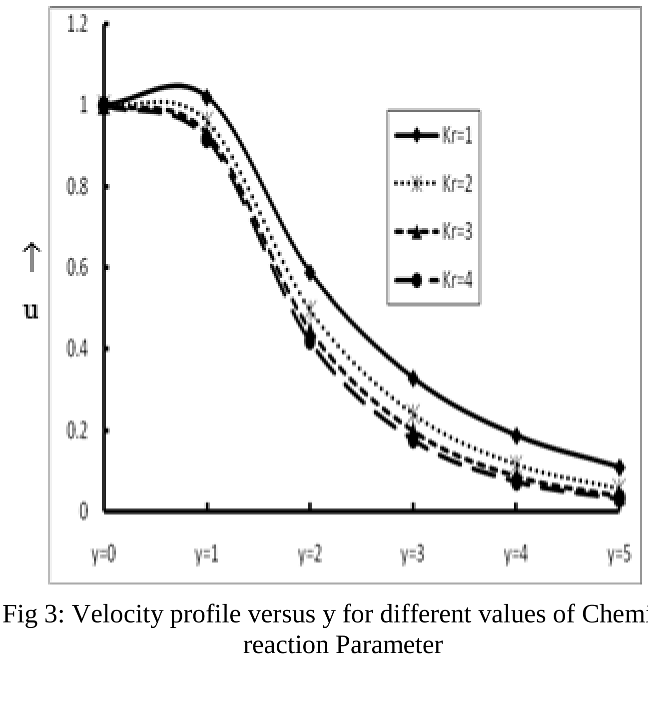

Fig 3: Velocity profile versus y for different values of Chemical reaction Parameter

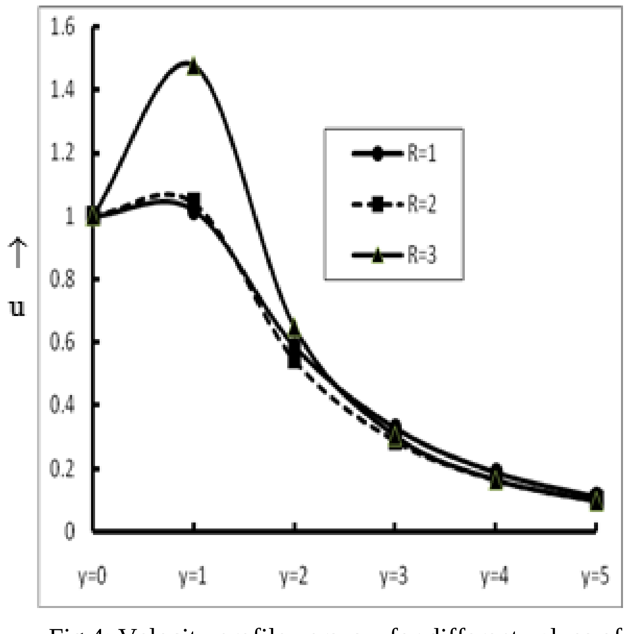

Fig 4: Velocity profile versus y for different values of Radiation Parameter

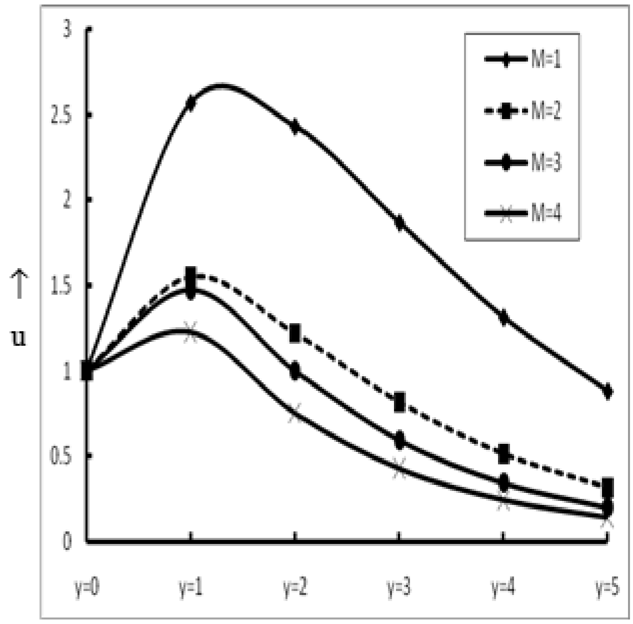

Fig 5: Velocity profile versus y for different values of Hartmann number

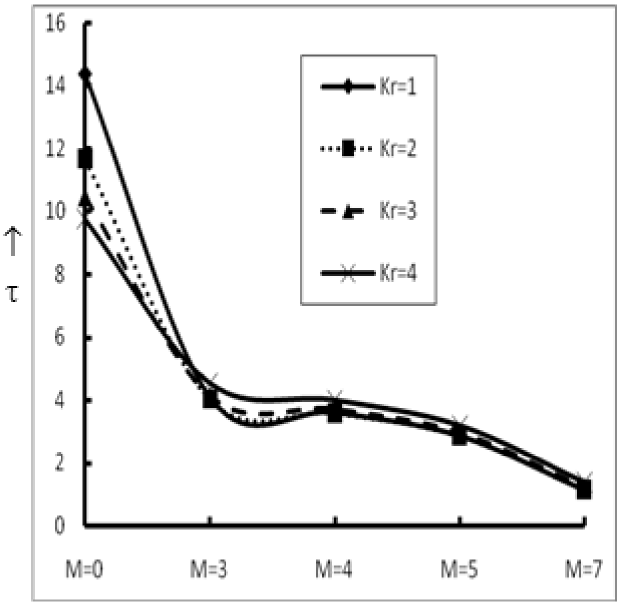

Fig 6: Skin Friction versus Hartmann number for different values of Chemical Reaction parameter

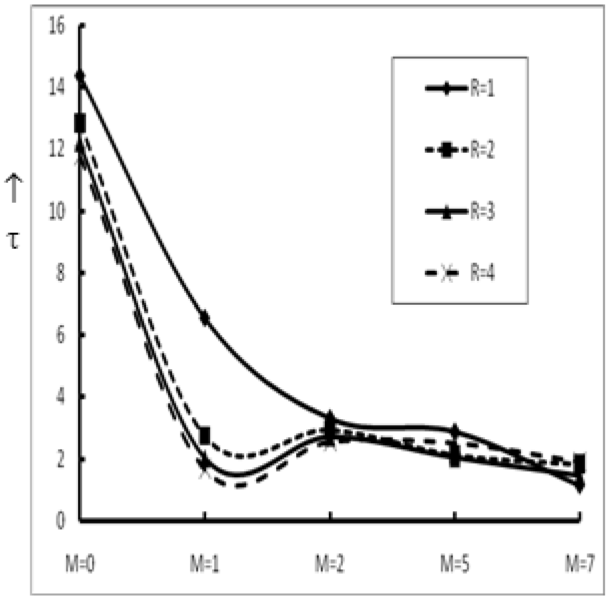

Fig 7: Skin Friction versus Hartmann number for different values of Radiation parameter

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